1 Important Constants & Equations

This week you covered the mathematics of measuring an electric field. The most complicated example involved an imaginary, purely mathematical surface: the Gaussian surface.

The amount of a uniform electric field passing through a surface, or the **electric flux**, is defined as:

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos(\theta) \qquad \text{(Uniform Electric Flux Through a Surface)}$$

The general electric flux through a closed surface:

$$\oint \vec{E} \cdot d\vec{A} \qquad \text{(General Electric Flux Through a Surface)}$$

The $\oint$ indicates that the surface is closed, meaning there are no holes or discontinuities.

As an expansion upon the General Electric Flux Through a Surface, Gauss's Law states:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \qquad \text{(Gauss's Law)}$$

Though charged conductors do not have an electric field *inside* them, at their surface, the electric field at the surface is:

$$\vec{E}_{\text{surface}} = \frac{\eta}{\epsilon_0}, \text{ perpendicular to the surface} \qquad \text{(Electric Field at the Surface of a Charged Conductor)}$$

The following constants are important for the study of electricity:

$e = 1.60 \times 10^{-19}$ C (Fundamental Unit of Charge)
$m_p = 1.67 \times 10^{-27}$ kg (Mass of Proton)
$m_e = 9.11 \times 10^{-31}$ kg (Mass of Electron)
$K = 8.99 \times 10^{9}$ Nm^2/C^2 (Electrostatic Constant)
$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 (Permittivity Constant)

2 Problems

1. There are three primary forms of symmetry you will deal with in electricity & magnetism.

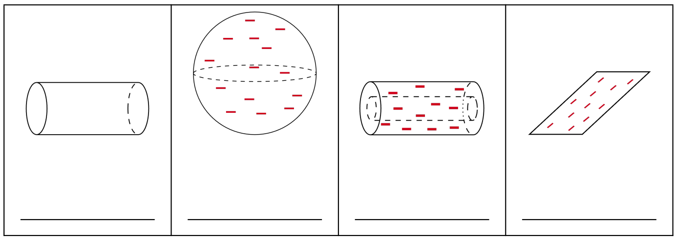
   (a) What are the three primary forms?

   i. ______________________

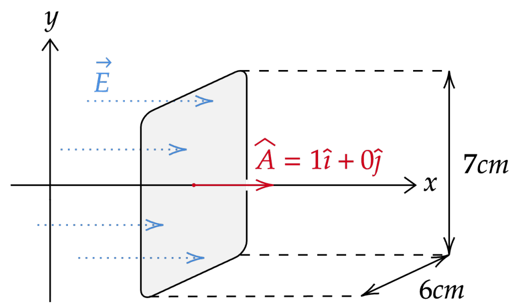
   ii. ______________________

   iii. ______________________

   (b) Identify the symmetry form associated with the following charge distributions:



2. As shown in the diagram below, a square loop with area A rests on the x-axis. Assume the area vector $\vec{A}$ points in the positive x-direction. Use the diagram to answer the following questions.



(a) Find the area vector of the square loop. (Recall that the area vector is the magnitude of the area times the unit area vector $\hat{A}$ (*shown in diagram*).)

$\vec{A} =$

(b) Find the electric flux Φ_e if $\vec{E} = (15\hat{\imath} + 0\hat{\jmath})$ N/C through the entire loop surface.

(c) Find the electric flux Φ_e if $\vec{E} = (10\hat{\imath} - 5\hat{\jmath})$ N/C through the entire loop surface.

(d) **(BONUS).** Find the electric flux if $\vec{E} = (15\,|y|\,\hat{\imath} + 5\hat{\jmath})$ N/C through the entire loop surface. (Hint: Let $d\vec{A} = \hat{A}dy$)

3. Assume there is a ball of charge with a radius r.

(a) Diagram this scenario.

(b) Find the electric field at a distance $R > r$ from the center of the ball of charge.

(c) Assume the ball has a uniform charge. Find the electric field at a distance $R \leq r$ from the center of the ball of charge.

4. **(CHALLENGE PROBLEM).** You work for CERN, the operator of world's largest particle accelerator. Scientists have asked you to propel an electron into a charged surface in hopes of learning something highly classified. While you may never know what it is they want to find out, you are given 100 grams of Aluminum to collide the electron into.

   (a) Assume that for each gram of Aluminum, a charge of $-5\mu C$ can be produced. What is the charge density ρ for the Aluminum (in $^{\mu\text{C}}/_{\text{cm}^3}$)? The density of Aluminum is 2.7 $^{\text{g}}/_{\text{cm}^3}$.

   $\rho =$

   (b) You decide to create a ball of aluminum because it has some nice symmetry. You wish to create an electric field strength of 27000 $^{\text{N}}/_{\text{C}}$ at *the surface* of the ball of Aluminum. What would the radius r of this ball of charge have to be? (Hint: Use a relationship between the charge density ρ and the charge Q)

(c) Find an expression for the electric field at a distance x from the surface of the Aluminum ball.

(d) Find the acceleration $a_e(x)$ of an electron (due to the influence of an electric field) at any point x from the surface of the Aluminum ball.

(e) **(CHALLENGE BONUS).** Using your answer in part (d), find an expression for the velocity $v_e(x)$ of the electron. (This one is quite challenging but give it a shot if you'd like!)